

5.2N Solving Quadratic Equations – Choosing the Best Method: Part I

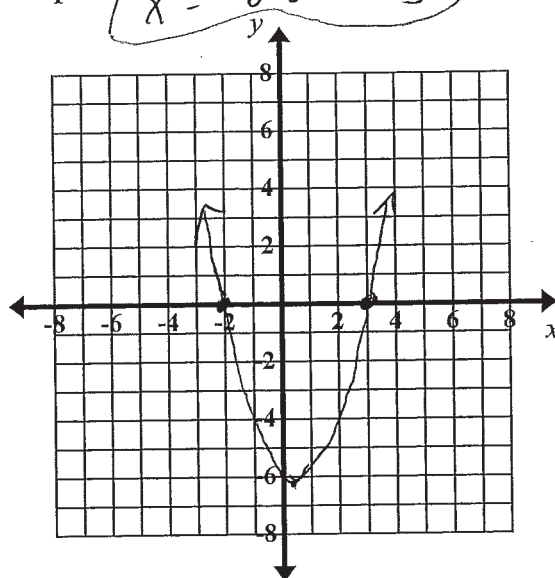
1. List 5 ways to solve a quadratic equation:

- Use a graphing utility to find real zeros
- Factor and use the zero product property
- Use square roots
- Complete the Square
- Use the Quadratic Formula

#2 – 3: Solve the following quadratic equations using the indicated methods.

2. $x^2 - x = 6$ $x^2 - x - 6 = 0$

Graph:



Factor:

$$x^2 - x = 6$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3 \text{ or } x = -2$$

Quadratic Formula:

$$a = 1$$

$$x^2 - x - 6 = 0$$

$$b = -1$$

$$c = -6$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{25}}{2} \rightarrow \begin{cases} \frac{1+5}{2} = 3 \\ \frac{1-5}{2} = -2 \end{cases}$$

Solution(s):

$$x = 3 \text{ or } x = -2$$

Which method was the most efficient for this problem and why?

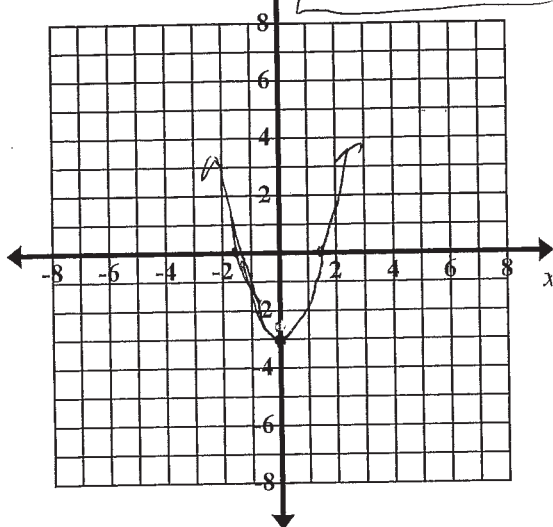
Factoring Method is the fastest and yields rational solutions safely.

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#2 – 3 (continued): Solve the following quadratic equations using the indicated methods.

3. $x^2 - 3 = 0$

Graph:



Square Root:

$$\begin{aligned}
 x^2 - 3 &= 0 \\
 \sqrt{x^2} &= \sqrt{3} \\
 |x| &= \sqrt{3} \\
 x &= \pm\sqrt{3}
 \end{aligned}$$

Quadratic Formula: $a=1$ $b=0$ $c=-3$

$$\begin{aligned}
 x^2 - 3 &= 0 \\
 x &= \frac{-0 \pm \sqrt{0^2 - 4(1)(-3)}}{2(1)}
 \end{aligned}$$

$$x = \frac{0 \pm \sqrt{12}}{2} = \frac{\pm 2\sqrt{3}}{2} = \pm\sqrt{3}$$

Solution(s):

$$x = \pm\sqrt{3}$$

Which method was the most efficient for this problem and why?

The SQUARE ROOT method
There was no x term, OR $b=0$

4. List the method(s) for solving quadratic equations that:

- You can *always* use but often it only gives *approximate* answers.
graphing calculator (but can't use to find complex/imaginary solns).
- You can *always* use to solve quadratic equations.
Completing the Square and Quadratic Formula
- You can only use *sometimes* to solve quadratic equations.
Factoring or Square Root

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#5 – 7: For each equation, determine an effective method for solving the quadratic equation and explain why you chose that method. Solve the quadratic equation with the method of your choice, keeping the answer exact (no decimal approximations).

5. $2x^2 + 5x - 3 = 0$

It factored easily.

$$(2x-1)(x+3) = 0$$

$$x = \frac{1}{2} \text{ or } x = -3$$

6. $-4x^2 + 4000x = 0$

It factored easily.

$$-4x(x-1000) = 0$$

$$x = 0 \text{ or } x = 1000$$

7. $x^2 + 7x - 18 = 0$

It factored easily.

$$(x+9)(x-2) = 0$$

$$x = -9 \text{ or } x = 2$$

#8 – 11: Determine an answer for each situation. Be sure to clearly record your thinking.

8. The product of two consecutive integers is 72. Find the two numbers.

Let $n = \text{any integer}$
 $n+1 = \text{Consec. int (larger)}$

$$n(n+1) = 72$$

$$n^2 + n - 72 = 0$$

$$(n-8)(n+9) = 0$$

$$n = 8 \text{ or } n = -9$$

$$n+1 = 9 \quad n+1 = -8$$

The 2nds are 8 and 9
 or -9 and -8

9. The length of a rectangle exceeds its width by 3 inches. The area of the rectangle is 70 square inches. Find its dimensions.

$$\begin{array}{|c|} \hline x+3 \\ \hline \end{array} \quad x \quad \begin{array}{|c|} \hline 70 \\ \hline \end{array}$$

$$x(x+3) = 70$$

$$x^2 + 3x - 70 = 0$$

$$(x+10)(x-7) = 0$$

$$x = -10 \quad x = 7$$

extraneous

$$\Rightarrow \begin{array}{|l|} \hline \text{width} = 7 \text{ in} \\ \hline \text{length} = 10 \text{ in} \\ \hline \end{array}$$

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 $(2x-1)(x+3)$
 $x = \frac{1}{2} \text{ or } x = -3$

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 $-4x(x-1000) = 0$
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 $n(n+1) = 72$
 $n^2 + n - 72 = 0$
 $(n-8)(n+9) = 0$
 $n = 8 \text{ or } n = -9$
 $n+1 = 9 \text{ or } n+1 = -8$
 The 2 #s are 8 and 9
 or -9 and -8

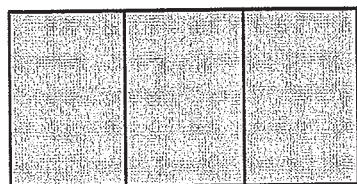
9. The length of a rectangle exceeds its width by 3 inches. The area of the rectangle is 70 square inches. Find its dimensions.

$x+3$
 $x \begin{array}{|c|} \hline 70 \\ \hline \end{array}$
 $x(x+3) = 70$
 $x^2 + 3x - 70 = 0$
 $(x+10)(x-7) = 0$
 $x = -10 \text{ or } x = 7$
 extraneous \Rightarrow width = 7 in
 length = 10 in

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#8 – 11 (continued): Determine an answer for each situation. Be sure to clearly record your thinking.

10. Suzie wants to build a garden that has three separate rectangular sections. She wants to fence around the whole garden and between each section as shown. The plot is twice as long as it is wide and the total area is 200 square feet. How much fencing does Suzie need?



2x

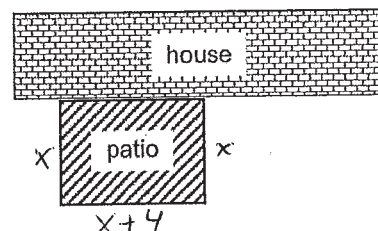
$$\begin{aligned} x(2x) &= 200 \\ 2x^2 &= 200 \\ \sqrt{x^2} &= \sqrt{100} \\ |x| &= 10 \\ x &= \pm 10 \\ \text{width } 10 \\ \text{length } 20 \end{aligned}$$

$$\begin{aligned} \text{Fencing} &= \text{Perimeter} + 2x \\ &= 2L + 2W + 2x \\ &= 2(2x) + 2(x) + 2x \\ &= 8x \\ &= 8(10) \\ \text{Fencing} &= 80 \text{ feet} \end{aligned}$$

11. Mike wants to fence three sides of a rectangular patio that is adjacent to the back of his house. The area of the patio is 192 ft² and the length is 4 feet longer than the width. Find how much fencing Mike will need.

$$\begin{aligned} x(x+4) &= 192 \\ x^2 + 4x - 192 &= 0 \\ (x-12)(x+16) &= 0 \\ x &= 12 \text{ or } x = -16 \\ &\quad \text{extraneous} \end{aligned}$$

$$\begin{aligned} \text{Fencing} &= x + (x+4) + x \\ &= 3x + 4 \\ &= 3(12) + 4 \\ \text{Fencing} &= 40 \text{ feet} \end{aligned}$$



#12 – 13: Solve each quadratic equation 2 different ways. Record the method that you are using for each.
(Square Roots – Factoring – Completing the Square – Quadratic Formula)

12. $2x^2 + 3 = 21$

Method 1: Square Roots

$$2x^2 + 3 = 21$$

$$2x^2 = 18$$

$$\sqrt{x^2} = \sqrt{9}$$

$$|x| = 3$$

$$x = \pm 3$$

Method 2: Factoring

$$2x^2 + 3 = 21$$

$$2x^2 - 18 = 0$$

$$2(x^2 - 9) = 0$$

$$2(x+3)(x-3) = 0$$

$$x = -3 \text{ or } x = 3$$

Solution(s): $x = \pm 3$

Square Roots because there was no x -term.

Factoring since difference of squares pattern is easy.

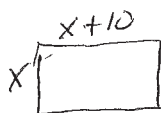
Why did you choose the two methods that you did and which do you feel is more efficient?

Square Roots has less chance for a careless sign error. Also, students have been solving with square roots longer than they have been factoring.

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#12 – 13 (continued): Solve each quadratic equation 2 different ways. Record the method that you are using for each. (Square Roots – Factoring – Completing the Square – Quadratic Formula)

13. The length of a rectangular pool is 10 meters more than its width. The area of the pool is 875 square meters. Find the dimensions of the pool.



Method 1: Completing the Square

$$\begin{aligned} x(x+10) &= 875 \\ x^2 + 10x + 25 &= 875 + 25 \\ \sqrt{(x+5)^2} &= \sqrt{900} \\ |x+5| &= 30 \\ x+5 &= \pm 30 \\ x &= -5 \pm 30 \end{aligned}$$

← 25
← 35 extraneous

width = 25 m
length = 35 m

Solution(s): width = 25 m
length = 35 m

Complete the square already had the constant isolated and it was easy to find $(\frac{b}{2})^2$. Factoring was nice since $a=1$.

Method 2: Factoring

$$\begin{aligned} x(x+10) &= 875 \\ x^2 + 10x - 875 &= 0 \\ (x-25)(x+35) &= 0 \\ x &= 25 \quad x = -35 \\ x+10 &= 35 \quad \text{extraneous} \end{aligned}$$

width = 25 m
length = 35 m

Why did you choose the two methods that you did and which do you feel is more efficient?

Complete the square was more efficient. Since $(\frac{b}{2})^2$ was a whole #, plus the square root property is well ingrained. Factoring might have taken longer to find 2 integers with a product of -875.

#14 – 17: Solve the following quadratic equations with the method of your choice. Verify that the answer is correct:

14. $3x^2 + 6x = -10$

$$3x^2 + 6x + 10 = 0$$

$$a=3 \quad b=6 \quad c=10$$

Use Quad. Formula:

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(3)(10)}}{2(3)}$$

$$x = \frac{-6 \pm \sqrt{-84}}{6}$$

$$x = \frac{-6 \pm 2i\sqrt{21}}{6} = \frac{-3 \pm i\sqrt{21}}{3}$$

✓ Verify that your answer(s) are solution(s):

$$\begin{aligned} 3\left(\frac{-3+i\sqrt{21}}{3}\right)^2 + 6\left(\frac{-3+i\sqrt{21}}{3}\right) &= -10 \quad \text{and} \quad 3\left(\frac{-3-i\sqrt{21}}{3}\right)^2 + 6\left(\frac{-3-i\sqrt{21}}{3}\right) = -10 \\ 3\left(\frac{9-6i\sqrt{21}-21}{9}\right) + 2(-3+i\sqrt{21}) &= -10 \\ \frac{-12-6i\sqrt{21}}{3} + 2(-3+i\sqrt{21}) &= -10 \\ -4-2i\sqrt{21}-6+2i\sqrt{21} &= -10 \\ -10 &= -10 \checkmark \end{aligned}$$

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#14 – 17 (continued): Solve the following quadratic equations with the method of your choice. Verify that the answer is correct:

15. $-3x^2 + 12x + 1 = 0$

$a = -3$ $b = 12$ $c = 1$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(-3)(1)}}{2(-3)}$$

$$= \frac{-12 \pm \sqrt{156}}{-6}$$

$$x = \frac{-12 \pm 2\sqrt{39}}{-6} \Rightarrow x = \frac{6 \pm \sqrt{39}}{3}$$

✓Verify that your answer(s) are solution(s):

$$-3\left(\frac{6 + \sqrt{39}}{3}\right)^2 + 12\left(\frac{6 + \sqrt{39}}{3}\right) + 1 = 0$$

$$-3\left(\frac{36 + 12\sqrt{39} + 39}{9}\right) + 4(6 + \sqrt{39}) + 1$$

$$\frac{75 + 12\sqrt{39}}{-3}$$

$$-25 - 4\sqrt{39} + 24 + 4\sqrt{39} + 1 = 0$$

$$0 = 0 \checkmark$$

16. $x^2 + 6x + 9 = 0$

$(x+3)(x+3) = 0$

$x = -3$

Also $-3\left(\frac{6 - \sqrt{39}}{3}\right)^2 + 12\left(\frac{6 - \sqrt{39}}{3}\right) + 1 = 0$

$$-3\left(\frac{36 - 12\sqrt{39} + 39}{9}\right) + 4(6 - \sqrt{39}) + 1$$

$$\frac{75 - 12\sqrt{39}}{-3}$$

$$-25 + 4\sqrt{39} + 24 - 4\sqrt{39} + 1 = 0$$

$$0 = 0 \checkmark$$

✓Verify that your answer(s) are solution(s):

$$(-3)^2 + 6(-3) + 9 = 0$$

$$9 - 18 + 9$$

$$0 = 0 \checkmark$$

17. $81x^2 + 1 = 0$

$$\frac{81x^2}{81} = \frac{-1}{81}$$

$$\sqrt{x^2} = \sqrt{\frac{-1}{81}}$$

$$|x| = \frac{i}{9}$$

$$x = \pm \frac{i}{9}$$

✓Verify that your answer(s) are solution(s):

$$81\left(\frac{i}{9}\right)^2 + 1 = 0$$

$$81\left(\frac{-1}{81}\right) + 1$$

$$-1 + 1 = 0 \checkmark$$

$$81\left(\frac{-i}{9}\right)^2 + 1 = 0$$

$$81\left(\frac{(-1)(-1)(i)(i)}{81}\right)$$

$$81\left(\frac{i^2}{81}\right)$$

$$81\left(\frac{-1}{81}\right) + 1 = 0 \checkmark$$

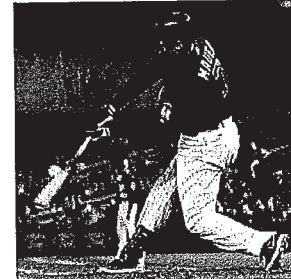
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18. The height h (in feet) above the ground of a baseball depends upon the time t (in seconds) it has been in flight. Joe takes a mighty swing and hits a bloop single whose height is described approximately by the equation $h = 80t - 16t^2$.

a) How long is the ball in the air?

$$\begin{aligned} -16t^2 + 80t &= 0 \\ -16t(t - 5) &= 0 \\ t = 0 \text{ or } t = 5 \text{ seconds} \end{aligned}$$

extraneous



b) When does the ball reach its maximum height? Use Graphing Calc.
After 2.5 seconds

c) What is the maximum height?

100 ft

- d) It takes approximately 0.92 seconds for the ball to reach a height of 60 feet. On its way back down, the ball is again 60 feet above the ground. What is the value of t when this happens?

Add the line $y = 60$ and use calculate intersect to get

$$t \approx 4.08 \text{ seconds}$$

Section 5.2N

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